Grade 11/12 Math Circles

October 4, 2023

Digital Signal Processing - Solutions

Exercise 1

Consider the continuous-time signal defined by $x(t) = t$, $0 \leq t \leq 12$.

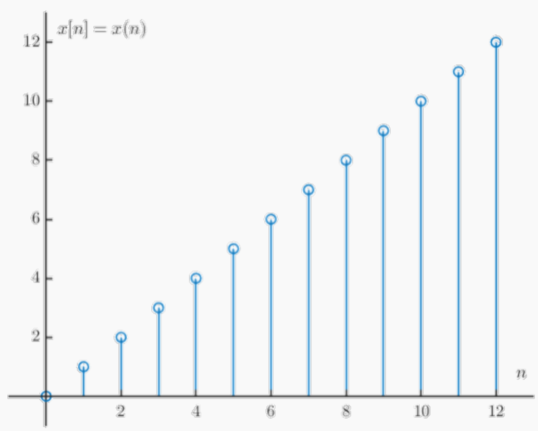
Compute and sketch the discrete-time (digital) signal $x[n]$ with sampling intervals:

a) $T = 1$, and

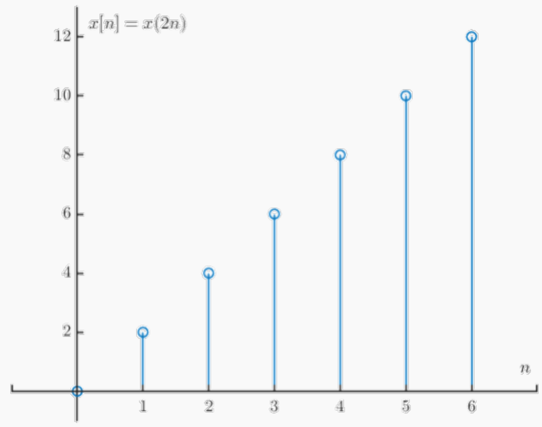
b) $T = 2$.

Exercise 1 Solution

a) $x[n] = x(n)$.

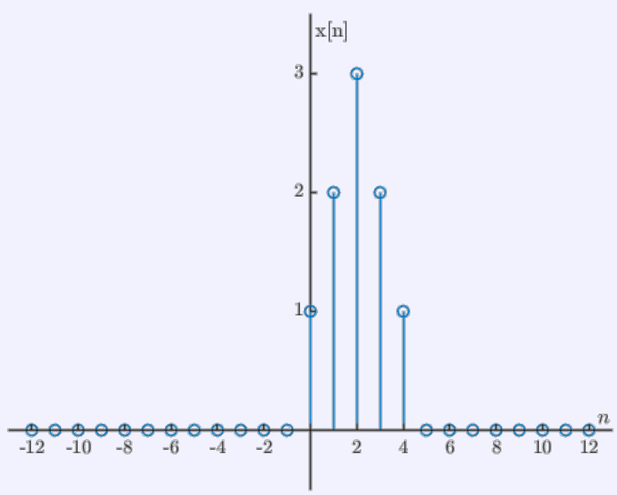


b) $x[n] = x(2n)$.



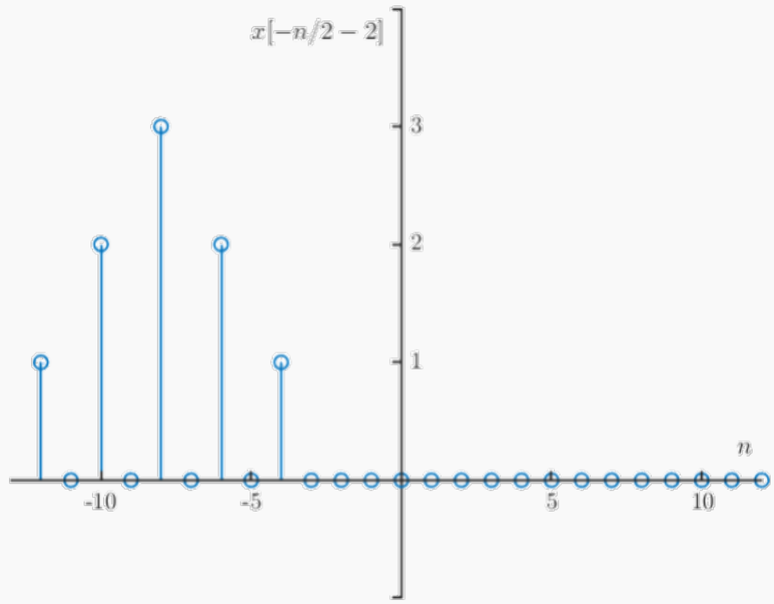
Exercise 2

Let $x[n]$ be the following signal.



Sketch the transformed signal $y[n] = x[\frac{-n}{2} - 2]$ by shifting, flipping, and then scaling the original signal. It may be helpful to sketch each intermediate step.

Exercise 2 Solution



Exercise 3

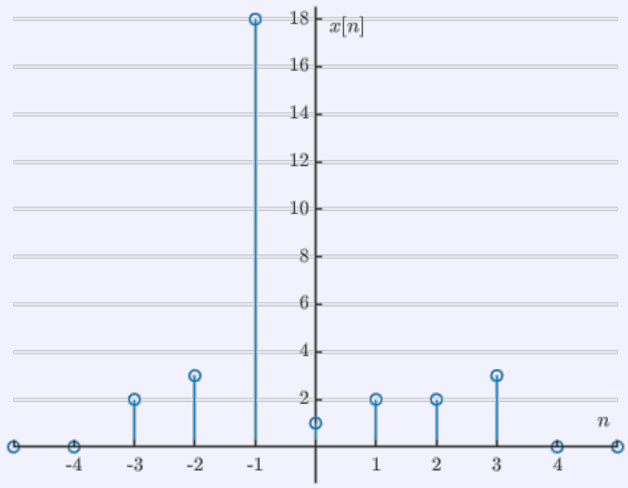
Write the delta function, $\delta[n]$, in terms of a sum (or difference) of shifted unit step functions.

Exercise 3 Solution

$\delta[n] = u[n] - u[n-1]$.

Exercise 4

Let $x[n]$ be the following signal.



Sketch the result of

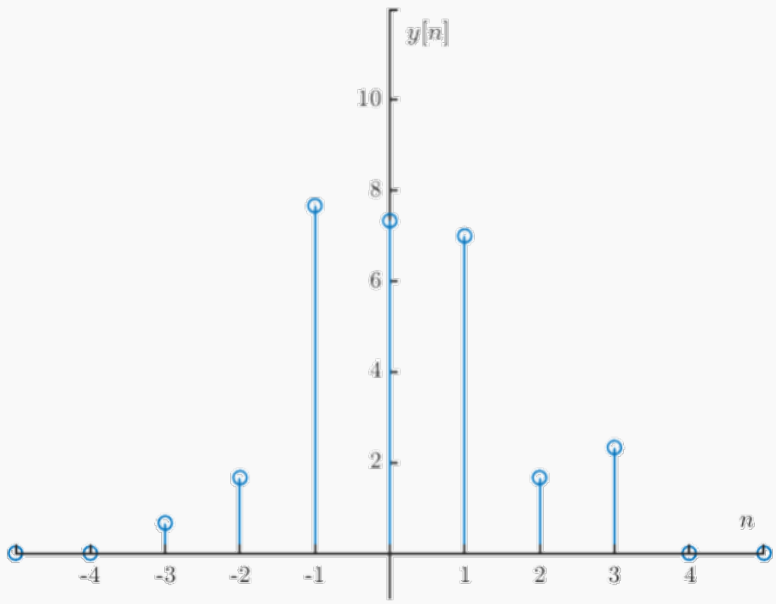
a) applying the moving average filter to $x[n]$ with $N = 3$, and

b) applying the median filter to $x[n]$ with $N = 3$.

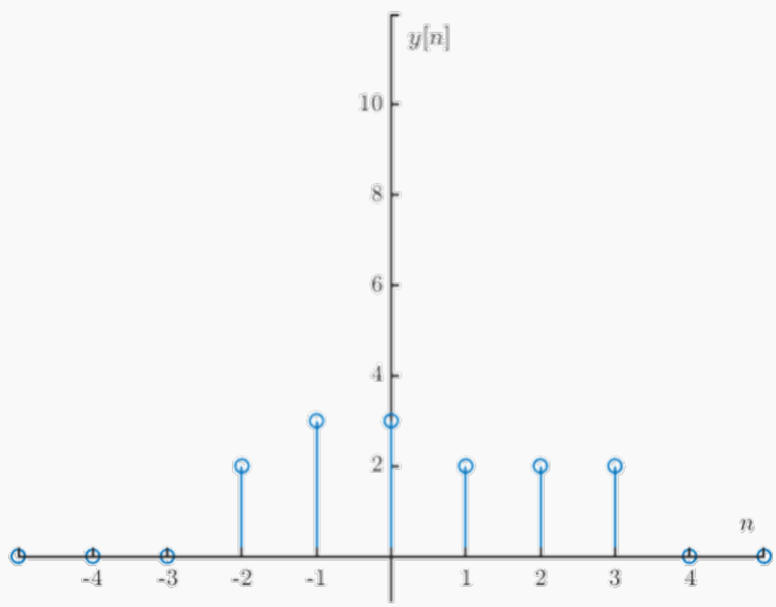
What differences do you notice?

Exercise 4 Solution

a) Moving average filter:



b) Median filter:



Exercise 5

An exponential moving average filter is defined by

$$y[n] = \alpha x[n] + (1-\alpha)y[n-1]$$

where $y[n]$ is the current output, $y[n-1]$ is the previous output, and $x[n]$ is the current input. The parameter α is a number between 0 and 1.

Assuming that $x[n] = 0$ when $n < 0$,

a) Find an expression for the output $y[n]$ in terms of only the previous input values, i.e. the values $x[n-k]$.

b) Using your result from a), determine the impulse response of the exponential moving average filter.

Exercise 5 Solution

a) We can determine $y[n]$ by substituting the previous inputs into the definition, as follows:

$$\begin{aligned} y[n] &= \alpha x[n] + (1-\alpha)y[n-1] \\ &= \alpha x[n] + (1-\alpha)(\alpha x[n-1] + (1-\alpha)y[n-2]) \\ &= \alpha x[n] + (1-\alpha)(\alpha x[n-1] + (1-\alpha)((\alpha x[n-2] + (1-\alpha)y[n-3])) \\ &\dots \\ &= \alpha \sum_{k=0}^{n} (1-\alpha)^k x[n-k]. \end{aligned}$$

b) The impulse response is the output of the filter when the input is a delta function. Using our result from a), we find that

$$\begin{aligned} h[n] &= \alpha \sum_{k=0}^{n} (1-\alpha)^k \delta[n-k] \\ &= \alpha(1-\alpha)^n. \end{aligned}$$